Foundations of Mathematical Logic
0421202001

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03/18/2022

Class Hours: Thu 1:15 –3:40 PM
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Course Description

Logic is traditionally regarded as a theory of reasoning, or, more specifically, the study of the principles used in the justification of a logically certain conclusion from a series of statements used in support of the claim. This process is structured in terms of objects we call arguments, which form the basic building brick of philosophy, mathematics, and computer science.

When arguments are given to us in the wild, however, they are not always given in an explicit way in which it is easy to isolate its parts. Such arguments are often obscured by the expressive aspects of the natural language and this prompts the idea of formalization, the process of making the structure of linguistic expressions explicit by translating them into a formal language, an abstraction of the natural language aimed at minimizing ambiguity.

Mathematical logic is the study of formal logic as a branch of mathematics. This course serves as an introduction to the foundations of mathematical logic. As such it covers fundamental concepts with emphasis on elementary set theory, formal languages and grammars, truth tables, natural deduction, proof normalization, first-order semantics, classical propositional and predicate logic, soundness and completeness, compactness and Löwenheim-Skolem, quantifier elimination, prenex normal forms, Herbrand's theorem, the theory of recursive functions, Peano arithmetic, and Gödel's incompleteness results.

Required Materials

This course is roughly based on the following textbook:

- Van Dalen, Dirk. *Logic and Structure*
  ISBN: N 978-3-540-20879-2

Any particular edition can be used for this course. Additional readings may be assigned during the semester and made available electronically available in PDF format.

Course Objectives

Upon the successful completion of this course, you will be able to:

1. read and give rigorous mathematical definitions;
2. understand and carry out formal proofs of mathematical theorems;
3. have a solid foundation to pursue further study of mathematical logic;

Prerequisites

Basic knowledge of mathematics such as elementary set theory is desirable, but not necessary. If you think such concepts can be challenging for you, it would be wise to devote some extra time to the course and practice your skills with more exercises from the book. If you are still having trouble keeping up with the classes, please talk to me. Learning logic can be a really fun experience and I want you to enjoy taking this course.

Assessment and grades

There will be only one exam at the end of the summer term. It will test on all the material that was taught up until two weeks prior to the exam. The questions will be roughly based on the questions from the homework assignments. Detailed information about the format and content of the exam will be provided as it approaches. To recapitulate the main results studied through the lectures, one review section will be given one week before the exam.

Homework

You will be assigned six homework assignments throughout the lectures. Learning logic is like learning a new language or a new skill like swimming or a musical instrument. It takes a lot of effort and daily practice. If you don't practice the new skill, you lose it.

Each homework assignment must be completed and turned in on time. If you missed the deadline because you were ill or for some other valid reason, please send me an email. I wish to evaluate your performance based on your efforts, so your homework should reflect your own work. Both handwritten and printed assignments are equally acceptable. For students trying to pursue further studies in logic, using LaTeX is highly encouraged.

Attendance policy

You are expected to attend every lecture and be on time. If you cannot come to class due to an emergency please let me know as soon as possible. If you miss a class it is your responsibility to make up the material missed and catch up with your classmates.

Feedback

I welcome feedback, be it positive or negative. If you wish, you can do this by speaking to me directly after class, sending me an email, or, if you prefer, sending me an anonymous note. Giving feedback will not affect your grade, neither positively nor negatively, but it will help me to see my lectures from different angles and develop new ways of improving them.

Schedule

The schedule is tentative and subject to change with fair notice. The items below should be viewed as the key concepts you should grasp in that week and thus can be used as a study guide before each exam.

Week 01, 02/24: Introduction to formal logic

- Overview of set theory
- Formal languages and grammar

Week 02, 03/03: Truth tables

- Truth functionality
- Validity and satisfaction

Week 03, 03/10: Equivalences and substitution

- Logical equivalences
- Conjunctive and disjunctive normal forms
- Functional completeness and interdefinability of connectives

Week 04, 03/17: Proof systems I

- Semantic consequence
- Hilbert-style axiomatic systems

Week 05, 03/24: Proof systems II

- Natural deduction
- Introduction and Elimination rules
- Proof normalization

Week 06, 03/31: Soundness and completeness

- The soundness theorem
- The completeness theorem

Week 07, 04/07: Predicate logic

- Predicates and relations
- Universal and existential quantifier
- Interdefinability of quantifiers
- Formal language and grammar

Week 08, 04/14: First order semantics

- Structures
- Validity, satisfaction, models
- Common properties of predicate logic

Week 09, 04/21:

- The language of identity, posets, groups, rings
- Peano arithmetic

Week 10, 04/28: Proof systems

- Hilbert-syle axiomatic systems
- Natural deduction
- Proof normalization

Week 11, 05/05: Soundness and completeness

- The soundness theorem
- The completeness theorem

Week 12, 05/12: Completeness and Applications I

- The compactness theorem
- Löwenheim-Skolem
- Decidability

Week 13, 05/19: Completeness and Applications II

- Quantifier elimination
- Prenex normal forms
- Herbrand's Theorem

Week 14, 05/26: Gödel's incompleteness theorems I

- The Entscheidungsproblem
- Primitive recursive functions
- Partial Recursive Functions

Week 15, 06/02: Gödel's incompleteness theorems II

- Revisiting Peano arithmetic
- Gödel encoding
- Undecidability of Peano arithmetic

Week 16, 06/09: Review session

Week 17, 06/16: Final exam